Technical Note

A Comparative Study on PSD Models for Chromite Ores Comminuted by Different Devices

Adem Tas¸demir*, Tuba Tas¸demir*

(Received: 14 August 2008; accepted: 12 March 2009)

DOI: 10.1002/ppsc.200800035

Abstract

The objective of this study was to characterize the particle size distributions (PSDs) of chromite ores comminuted by different devices, i.e., subjected to different breakage modes and to compare the performances of the PSD functions selected. Different PSDs were obtained for five different mineralogical samples of chromite ores by jaw, cone and hammer crushing and ball mill grinding. The PSDs of the products were characterized to find the most suitable model by Gates–Gaudin–Schuhmann (GGS) and Rosin–Rammler (RR) functions. It was found that the PSDs of all chromite types in single-pass devices, i.e., jaw and cone crushing were better described by the GGS model than RR model. The RR model gave the best results for all ore sample PSDs generated by retention type systems, i.e., hammer crushing and ball mill grinding. Both distribution functions gave higher $R^2$ values as the size distribution became uniform. The results of piecewise regression were found very useful to improve the performance of GGS distribution in terms of correlation coefficients for samples from a hammer crushe and ball mill.

Keywords: comminution, Gates–Gaudin–Schuhmann distribution, particle size distribution, piecewise linear regression, Rosin–Rammller distribution

1 Introduction

Particles must be characterized in terms of particle size and size distribution for a variety of reasons [1], including that the behavior and properties of particulate materials are, to a large extent, dependent on particle size, size distribution and shape [2]. Particle size is probably the single most important physical characteristic of powders. Particle size distribution (PSD) is frequently examined to obtain details on the particulate material. A fundamental knowledge of particle size and distribution is essential for a wide variety of industrial processes [3]. This is valuable in the production of particles of specific sizes to control process efficiency and product quality. Knowledge of the distribution of particulate material properties such as size, density, hydrophobicity, magnetic susceptibility, etc., is important in many industrial applications including cement, food, pharmaceuticals, cosmetics, pigment, fertilizers and mineral processing [4]. It influences the combustion efficiency of pulverized coal, the setting time of cements, the flow characteristics of granular materials, the compacting and sintering behavior of metallurgical powders and the masking power of paint pigment [5]. Transport and chemical reactivity problems are also functions of particle size and PSDs. A fundamental building block in the development of engineered solutions to mineral processing problems is the ability to readily determine and to mathematically describe PSDs [6]. Most mineral processing operations rely on the measurement of size distributions, as this is a key factor in improving process efficiencies. Mineral manufacturing processes often involve comminution.

* Dr. A. Taşdemir (corresponding author), Dr. T. Taşdemir, Eskişehir Osmangazi University, Department of Mining Engineering, Mineral Processing Division, 26480, Eskişehir (Turkey).
E-mail: atasdem@ogu.edu.tr

© 2009 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim
http://www.ppsc-journal.com
Amongst other uses, knowledge of particle size is used in the development of sampling protocols which in turn feeds into quality control, whereas PSD parameters are employed in modeling and simulation of various unit operations, especially in comminution and in physical enrichment separations. Models described for plant equipment, such as cyclones or flotation cells, require a knowledge of particle size as an input. The separation by size is, of course, another frequently used unit operation, and here too, knowledge of particle size allows determination of the effectiveness of such separations [6].

Particle size, like other variables in nature, tends to follow well-defined mathematical laws in its distribution. The size distribution of ground materials is typically skewed and normal distribution for ground materials is uncommon and occurs only for narrow size ranges [2]. Numerous equations have been developed to describe fragment generation mathematically after comminution processes [7]. Several two-parameter mathematical models and expressions have been published, ranging from the well known normal and log-normal, to the Rosin–Rammler (RR) and Gates–Gaudin–Schuhmann (GGS) models; the most important grain-size distribution functions have been reviewed by Allen [2] and King [8]. Although numerous three- and four-parameter models have also been proposed for greater accuracy in describing PSDs, their widespread application has been limited due to their greater mathematical complexity [5].

The results of PSD analysis may be expressed in terms of cumulative percent oversize or undersize in relation to the diameters of the particles, or as a distribution of the amounts present in each of a defined number of size classes [9]. In many applications, particle-size results are processed by plotting the cumulative frequency data on a scale expressed by the related distribution model. If a straight line is obtained, the particle-size distribution is said to obey the distribution function.

The characteristics of a PSD generated by comminution can be dependent on a variety of factors such as the extent of comminution mechanisms applied by the devices, comminution conditions, initial ore characteristics, whether a size classification system is applied during comminution or not, etc. Comminution conditions that favor one breakage mode over another may be critical in determining the PSDs of products. For any comminution system, the prevailing conditions generally depend on the type of comminution device and the initial properties and size of the particles being broken [10]. In comminution, different fracture mechanisms exist. These do not occur alone, but are found in combination with one another. Only their relative predominance varies as a function of the machine type, the operating conditions and the material being ground [11]. Altering the grinding environment in a single device may change the relative proportion of dominant breakage events, and consequently alter the PSD of the ground product [12].

No fundamental mechanism or model enables a theory on particle-size distribution to be built. As a result, a wide variety of empirical models or equations have been developed to characterize experimental PSDs of comminuted products [13]. The two most common methods, which are often applied to comminution studies, are the GGS and RR models. Both methods were derived from attempts to represent PSD curves by means of equations which result in scales which, relative to a linear scale, are expanded in some regions and contracted in others [14]. The various equations fit specific situations and no universal equation has been accepted yet [15]. In the literature section of a study, Buchholtz et al. [16] revealed that in some situations the RR law proved superior whereas in other cases the GGS law was more adequate.

In this study, different PSDs showing different physical properties were obtained for five different mineralogical samples of chromite ores comminuted under the usual working conditions for the different devices. The samples were subjected to different comminution mechanisms by jaw crusher, cone crusher, hammer crusher and ball mill and thus, different particulate products of differing size ranges were generated. To establish the suitability of the GGS and RR distribution functions in all cases, the accuracy of the functions in determining PSDs were tested to determine if they could be adopted in all situations. Also, piecewise linear regression was applied to improve the performance of the GGS model for hammer crusher and ball mill products by dividing the PSDs into two regions.

2 Materials and Methods

2.1 Mineralogical Characteristics of Chromite Ores Used

The five different mineralogical samples of chromite ores used in these experiments were taken from the run-of-mines, namely Bantli which was from Karaburhan, Eskisehir, Turkey and Dereboyu, Kef, Lasir and Yunuskuyu which were from Guleman, Elazig, Turkey. Chemical analysis of chromites by XRF is given in Table 1. Thin and polished sections from the lump samples of chromites were made for mineralogical examination. The unbroken grain size distributions were carried out on the cube-shaped polished sections of each ore sample by an automatic image analyzer and were found to give excellent fits to lognormal distribution [17]. These measurements were also used to designate the degree of cataclastic deformation of the chromite ores by a number-
based fractal approach proposed by Blenkinsop and Fernandes [18]. According to this evaluation, cataclastic deformation of the chromite ores studied was dominated by extension microstructure due to fragmentation by micro fracturing. The XRD studies were also carried out on the representative samples of chromite ores. The results of these detailed mineralogical examinations and unbroken grain size distributions have been published elsewhere [17]. Thus, the mineralogical properties of the chromite ores used in this study are summarized briefly below:

**Bantli Ore:** This banded ore type consisted dominantly of serpentinized olivines. The chromite grains were mainly of cataclastic texture due to tectonic effects. The fractured chromite crystals were filled with serpentines. The serpentine, which was of sieve texture, contained olivine relicts. **Dereboyu Ore:** This spotted ore type was composed mainly of a pyroxene mineral. The pyroxene was serpentinized in the contact zone of chromite grains and filled the fractures. The chromite grains were fractured and showed cataclastic texture. **Kef Ore:** This massive type of chromite consisted mainly of unaltered olivine minerals. The alteration products were serpentine and chlorite minerals. The chromite grains within the unaltered olivine kept their original shapes and were less fractured than the grains within the altered olivine. The fractured chromite crystals showed cataclastic structure. The chromite grains were less fractured than the other ore types. **Lasir Ore:** This was a disseminated type of chromite ore. The alteration products were serpentine as chlorite and serpentinized olivine. The chromite crystals were generally broken due to fragmentation effects to obtain cataclastic texture. **Yunuskuyu Ore:** The Leopard type of chromite ore comprised dominantly of serpentinized olivine minerals, which show sieve texture. The chromite grains in the nodular were fractured and showed cataclastic texture. The other altered minerals observed in the ore were chlorite and talc.

Both, the mineralogical examinations and quantitative grain size measurements indicated that the chromite ore samples were deformed differently by cataclasis and have different mineralogical properties.

### 2.2 Comminution Studies

Jaw, cone, and hammer crushers as well as ball mills were used in this study to comminute the ore samples. During the experiments, the comminution devices were operated under usual working conditions since the purpose of the study did not deal with analyzing the effect of operational parameters on the PSDs of chromite samples. The general flow sheet of comminution applied in this work is shown in Figure 1. The amount of comminuted samples of each ore from each device was reduced by applying the sampling procedures and these representative samples were used in sieve analysis. First, the chromite samples were crushed to –15 mm by a jaw crusher. The –15 +3.35 mm fraction was used for cone and hammer crusher feeds. After the first stage of crushing by the cone crusher, the product was sieved and the +3.35 mm fraction was crushed by the cone crusher again (second stage), making the output of the crusher narrower. A –2 +1 mm feed size was used for ball mill grinding. Ball mill conditions for the chromite minerals studied are given in Table 2. A loading of 20% of the mill volume filled by the ball bed and a fractional interstitial filling of the bed voids by the dry powder of 0.47 at 72.4% of the critical speed were chosen to run the

---

**Table 1: XRF analysis of chromite ores used.**

<table>
<thead>
<tr>
<th>Compound</th>
<th>Bantli</th>
<th>Dereboyu</th>
<th>Kef</th>
<th>Lasir</th>
<th>Yunuskuyu</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wt.%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MgO</td>
<td>30.0</td>
<td>18.5</td>
<td>20.0</td>
<td>20.0</td>
<td>28.0</td>
</tr>
<tr>
<td>Al₂O₃</td>
<td>3.1</td>
<td>9.9</td>
<td>17.0</td>
<td>9.7</td>
<td>3.3</td>
</tr>
<tr>
<td>SiO₂</td>
<td>23.2</td>
<td>9.3</td>
<td>7.4</td>
<td>14.1</td>
<td>24.3</td>
</tr>
<tr>
<td>CaO</td>
<td>0.3</td>
<td>0.5</td>
<td>0.4</td>
<td>1.1</td>
<td>0.3</td>
</tr>
<tr>
<td>Fe₂O₃</td>
<td>8.6</td>
<td>13.9</td>
<td>12.7</td>
<td>11.6</td>
<td>7.0</td>
</tr>
<tr>
<td>Cr₂O₃</td>
<td>25.0</td>
<td>45.2</td>
<td>39.8</td>
<td>41.3</td>
<td>30.3</td>
</tr>
<tr>
<td>LOI*</td>
<td>9.8</td>
<td>2.65</td>
<td>2.75</td>
<td>2.05</td>
<td>6.8</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

*All samples have Na₂O (%) = 0.1; P₂O₅ (%) = < 0.1; K₂O (%) = 0.1; TiO₂ (%) = 0.1; MnO (%) = 0.1.

* Loss of ignition

---

© 2009 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim

http://www.ppsc-journal.com
Table 2: Ball mill characteristics and experimental conditions.

<table>
<thead>
<tr>
<th>Mill</th>
<th>Inner diameter, D (mm)</th>
<th>Length, L (mm)</th>
<th>Volume, V (cm³)</th>
<th>Critical speed, Nc (rpm)</th>
<th>Operational speed</th>
<th>(72.4% of critical speed) (rpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Balls</td>
<td>Material</td>
<td>Ceramic</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mean diameter (mm)</td>
<td>24</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Specific gravity (g/cm³)</td>
<td>3.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total mass (g)</td>
<td>1272.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fractional ball filling, J</td>
<td>0.20</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Void between balls (cm³)</td>
<td>235.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>Feed size (mm)</td>
<td>2–1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Volume of samples (cm³)</td>
<td>110</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(a\); \(N_c = 420 / D^{1/2}\) (rpm); \(D\) in cm.

\(b\); \(J = (\text{mass of balls}/\text{ball density})/\text{mill volume})^{0.6}/0.6\)

\(c\); \([J^{*}/\text{mill volume}] - \text{[volume of balls]}\)

tests. The ore samples were ground until 80% of their undersize was minus 1 mm. This condition was achieved in 30 min for Kef ore, while the other ores took 20 min. Representative samples of particulate products generated in each device were classified by sieving. Thus, the PSDs of different particle size ranges were obtained after sieving the comminuted products of chromites. Retsch test sieve series including 12700, 9525, 4750, 3350, 2360, 2000, 1000, 800, 600, 500, 400, 300, 212, 150, 106, 75, 53 and 38 µm were used for sieving. All these sieves were used for sieving the jaw crusher, while the 4750–38 µm sieves were used for sieving after the first stage of cone crushing and hammer crushing. The 3350–38 µm and 1000–38 µm sieves were used for sieving after the second stage of cone crushing and ball milling, respectively. Particles greater than 1 mm were sieved by dry sieving while wet sieving was applied for particles less than 1 mm. All sieving studies were performed by manual sieving until there was no particle that would pass through a sieve.

### 2.3 GGS and RR Mathematical Models for PSDs

The Gates–Gaudin–Schuhmann (GGS) and Rosin–Rammer (RR) distributions were selected as they are widely utilized for to determine the size distributions of particulate materials after comminution. The GGS distribution is given as [19]:

\[ M = 100[(x/k)^a], \]

where \(x\) is the screen size, \(M\) is the cumulative mass (in %) passing size \(x\), \(k\) is the size parameter and \(a\) is the distribution parameter. Theoretically, lower values of \(a\) suggest more fines, more large particles and fewer particles in the middle range [20]. The higher the value of \(a\), the narrower the distribution.

If we take logs of both sides of this equation and rearrange it, then:

\[ \ln (M/100) = a \ln x + \ln c \text{ (constant)}. \]

This is the equation of a straight line if \(x\) and \(M\) are plotted on a log-log scale. The slope of the straight line will be the distribution parameter, \(a\), and the intercept of the straight line, when \(M\) equals 100 % will be the size parameter, \(k\) and it equals:

\[ k = \exp[(- \ln c)/a]. \]

The RR distribution is given as [19]:

\[ R = 100 \exp \left[ -\left( x/x' \right)^b \right]. \]

Eq. (4) can be rewritten as follows:

\[ 100/R = \exp \left[ \left( x/x' \right)^b \right]. \]

where \(x\) is the screen aperture size, \(R\) is the cumulative mass (in %) retained on size \(x\), \(x'\) is the size parameter, \(b\) is a measure of the spread of particle sizes. Small values of \(b\) indicate a scattered distribution, and large values imply uniform distribution [21].

The application of the function to a particular distribution and the calculation of its parameters are often done via linear regression of data represented as:

\[ \ln \ln (100/R) = b \ln x + \ln c \text{ (constant)}. \]

This is the equation of a straight line if \(\log (100/R)\) is plotted against \(x\) on a log-log scale. The parameters of the RR distribution, \(b\) and \(x'\) are obtained from the slope of the straight line and the intercept at the horizontal line at \(R = 36.79\%\) respectively. The \(x'\) can be calculated as:

\[ x' = \exp[(- \ln c)/b]. \]
2.4 Piecewise Linear Regression

The GGS distribution function in Eq. (2) was fitted as a two-parameter model \((a \text{ and } \ln c)\) within the whole domain of data availability. The following equation (Eq. (8)) suggested the existence of two power laws separated by a breaking point, \(\ln (x_c)\). This could mean that the log-transformed data would yield two straight lines separated by a cutoff, \(\ln (x_c)\). Thus, Eq. (8) was fitted as a five-parameter model \((a_1, a_2, \ln c_1, \ln c_2, \text{ and } \ln (x_c))\) using piecewise linear regression:

\[
\ln \left(\frac{M}{100}\right) = a_1 \ln x + \ln c_1 \ (\ln x \leq \ln x_c)
+ a_2 \ln x + \ln c_2 \ (\ln x > \ln x_c),
\]  (8)

where \(a_1\) and \(a_2\) are the slopes of the first and the second domain, respectively, and \(\ln (x_c)\) is the cutoff of the whole domain [22]. Each term in parenthesis represents a logical operation. This choice partitions the whole domain into two sub domains. The value of \(\ln (x_c)\) can be considered as the upper limit of the first domain and the lower cutoff of the second domain. The latter assumes that the upper limit of the second domain is the system size.

2.5 Data Analysis

The parameters from Eqs. (2), (6) and (8) were estimated by a Quasi-Newton non-linear method for fitting each data set. In order to compare the observed data with the distribution functions, some measure of the quality of fit between the functions and data is required. In this work, the coefficient of determination \((R^2)\) was used as a criterion in the selection of the appropriate fitting method. If \(R^2\) is close to unity, the distribution is a more appropriate fit to the experimental data. The parameters from the distribution functions were fitted to the experimental distributions in order to determine the most suitable PSD models for comminuted chromite samples with different mineralogical characteristics. Linear regression analyses between predicted and observed mass values using the PSD models (GGS model, RR model and a piecewise approach to GGS model) were also carried out. All the statistical analyses were computed using the Statistica software package [22]. The values of \(\ln (x_c)\) in Eq. (8) was estimated by the software to give the best breakpoints for the chromite ores.

3 Size Reduction Mechanisms in the Comminution Devices Used

It is well known that comminution in general is an energy intensive process. The energy required for breakage in comminution applications is dependent upon the physical properties of the material and the quantity of material being crushed. The rate of energy input is dependent upon the type of comminution device used since the application of the comminution force changes with machine type [23]. In this research, the chromite ore samples of different mineralogical characteristics were subjected to different predominant breakage mechanisms by different comminution devices. The forces acting on the particles can be classified into impact forces, compressive forces and abrasive forces in any comminution system. This classification is based on the direction and magnitude of the force acting on particles. In these systems, particles undergo impact, compression and abrasion repeatedly [24]. The mechanisms of size reduction during crushing and grinding are different. The chief difference being that in crushing operations the size reduction is more by compression and impact and less by attrition, while in grinding the forces of attrition are much greater in addition to impact forces [19]. The combination of the above types of fragmentation yields a product characteristic of the ground material and the grinding media, as each type is active to a different extent with different machines and materials [15]. Jaw and cone crushers are the most common types of primary compression crushers. These are essentially considered as single-pass devices since they offer limited retention time for the broken ores [10]. Each applies a compressive force to the rock particles as they come in contact with the crushing surfaces. The force is applied slowly (in comparison to impact machines) resulting in abrasion and cleavage fracture [23]. This produces fragments of slightly smaller size than the initial particles and also much finer particles [8]. Impact crushers such as hammer crushers apply a high-speed impact force to rock particles using hammers or blow bars. The rate of energy input is much higher causing particles to shatter [23]. In grinding, the combined action of repeated impact and abrasion over time causes size reduction to takes place [19]. Hammer crushers and ball mills can be considered retention type devices as the particles are subjected to repeated breakage [10]. This leads to a wide range of fragment sizes being produced, most being much smaller in size than the original particle. The difference between the finest and the coarsest size is small when the particles are subjected to a repeated fragmentation process [25].

4 Results and Discussion

Depending on the initial mineralogical properties, size of the ores, retention times in the devices and dominant
breakage modes applied by the different comminution devices, different PSDs with different ranges of particle sizes were obtained. The PSDs of chromites generated by similar dominant energy-events were evaluated together and the results obtained from the different mineralogical samples of chromite ores are presented in the following.

4.1 Characterization of PSDs of Jaw and Cone Crushers

The performance of the GGS and RR distribution models for jaw and cone crusher products is shown in Figure 2. Tables 3–5 show the model parameter values obtained when the RR and GGS functions were applied. These results suggested that initial mineralogical ore characteristics can be expected to affect the PSDs after comminution because there are different PSD model parameters for ore samples comminuted in the same device, as seen in Tables 3–5. Chromite ores showed a different size distribution modulus for GGS and RR models, a and b, which characterize the broadness of distributions. Bantli ore showed the widest size distributions compared to the other ores (lower values of a and b) and the differences in a and b between the other ores was quite small. It can be concluded that the size distribution of different mineralogical samples of chromites generated by jaw and cone crushers indicated that PSDs depended to a large extent on the mode of fragmentation used, which has an important influence on the particle size distribution of the resultant fragments, and to a lesser extent on the nature of the ores crushed. Because ores comminuted by the same devices showed a general characteristic distribution trend when the PSDs are shown on both, GGS and RR plots (Figure 2).

Figure 3 shows the relationship between the coefficient of determination of GGS and RR models for single-pass devices, i.e., jaw and cone crushers. In this figure, each ore samples is given a number and shown in the x axis. Since the dominant size reduction mechanism in jaw and cone crushers is compression, the effective breakage mode is mainly cleavage and abrasion. PSDs of minerals generated from these devices have more fines, more large particles and fewer particles in the middle range. The characteristic PSDs obtained for all differ-

---

**Table 3: PSD parameters of GGS and RR models for jaw crusher.**

<table>
<thead>
<tr>
<th>Ores</th>
<th>GGS predictions</th>
<th>RR predictions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a k R²</td>
<td>b x' R²</td>
</tr>
<tr>
<td>Bantli</td>
<td>0.67 25045 0.980</td>
<td>0.75 14700 0.962</td>
</tr>
<tr>
<td>Dereboyu</td>
<td>0.66 20130 0.990</td>
<td>0.75 11300 0.976</td>
</tr>
<tr>
<td>Kef</td>
<td>0.73 22770 0.990</td>
<td>0.81 13980 0.972</td>
</tr>
<tr>
<td>Lasir</td>
<td>0.71 15500 0.990</td>
<td>0.80 8930 0.986</td>
</tr>
<tr>
<td>Yunuskuyu</td>
<td>0.74 18930 0.982</td>
<td>0.82 11480 0.972</td>
</tr>
</tbody>
</table>

**Table 4: PSD parameters of GGS and RR models for the first stage of cone crusher.**

<table>
<thead>
<tr>
<th>Ores</th>
<th>GGS predictions</th>
<th>RR predictions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a k R²</td>
<td>b x' R²</td>
</tr>
<tr>
<td>Bantli</td>
<td>0.71 18830 0.958</td>
<td>0.77 12525 0.927</td>
</tr>
<tr>
<td>Dereboyu</td>
<td>0.79 12250 0.984</td>
<td>0.86 8310 0.954</td>
</tr>
<tr>
<td>Kef</td>
<td>0.81 13140 0.978</td>
<td>0.87 9280 0.958</td>
</tr>
<tr>
<td>Lasir</td>
<td>0.82 9425 0.986</td>
<td>0.90 6250 0.964</td>
</tr>
<tr>
<td>Yunuskuyu</td>
<td>0.78 12070 0.974</td>
<td>0.85 8050 0.949</td>
</tr>
</tbody>
</table>

---

Fig. 2: GGS and RR plots of PSDs of chromite samples comminuted by jaw and cone crushers.
ent mineralogical samples of chromite ores which were subjected to this kind of breakage mechanism were found to be represented better by the GGS function than the RR model. In GGS distribution, plotting on a log-log scale considerably expands the region below 50% in the cumulative undersize curve, especially below 25%. It does, however, severely contract the region above 50%, and especially above 75% [14].

### 4.2 Characterization of PSDs of Hammer Crusher and Ball Mill

The estimated distribution parameters of GGS and RR methods for the hammer crusher and ball mill are shown in Tables 6 and 7. In Figure 4, the cumulative mass fractions of the hammer crusher and ball mill are plotted as a function of particle diameter on GGS and RR distribution scales. The values of a and b depend on the mineralogical properties of ore samples since different values in the same devices are obtained from the different ores. This excludes the Bantli ore, which gives the most scattered and a wider size distribution with a smaller distribution modulus for both distribution models in the same devices. The differences in values between the other ores, which gave a more uniform distribution and hence higher values, are quite small. Although the Kef ore’s distribution modulus in the ball mill was quite small compared with the other ores, this

© 2009 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim http://www.ppsc-journal.com
was due to the higher retention time required to achieve a product which has \( d_{90} \) less than 1 mm. Here again, the results indicated that the parameters of particle size distribution functions are largely dependent upon the type of the comminution device and hence the energy events applied rather than the type of the ore.

The coefficients of determination of GGS and RR models for retention type devices, i.e., a hammer crusher and ball mill were plotted versus individual ore sample values and illustrated in Figure 5. The RR model can be suggested to describe the characteristic PSD curves of chromite ores from hammer crushing and ball milling, since it fitted the PSD data better than the GGS function for all types of chromites.

In comparison with the GGS model, the RR plot expands the regions below 25\% and above 75\% cumulative undersize and it contracts in the 30–60\% region [14]. Thus, the double log scale expands the fine and coarse ends of the size range and compresses the mid range.

### 4.3 Application of Piecewise Linear Regression

The plots of hammer crusher and ball mill GGS distribution show that a single power law could not describe the data across the entire range of measured particle sizes. Different power laws applied to the two domains in the GGS distribution plots of the hammer crusher and ball mill. A piecewise linear regression was applied to the GGS model of hammer crusher and ball mill by using Eq. (8). Figure 6 displays some representative PSD of the Lasir ore sample drawn on a double logarithmic scale for the hammer crusher and ball mill, respectively. The plots clearly show that a single power-law cannot describe the data across the entire range of measured particle sizes, but two power-law functions can be fitted across the data points, indicating two different scaling regimes for different ranges of particle sizes. It was noted that the transition point from the first linear region to the second linear region changes with the type of chromite sample (Table 8). Thus, the identified power-law domains separate the particle sizes into two classes and values of \( a \) have been estimated by linear fitting of

![Fig. 5: The \( R^2 \) values obtained from hammer crusher and ball mill products fitted by GGS and RR models.](image1)

![Fig. 6: Fit to the piecewise model to GGS distribution of Lasir ore for hammer crusher and ball mill product.](image2)

### Table 8: Piecewise regression model parameters for hammer crusher and ball mill.

<table>
<thead>
<tr>
<th>Ores</th>
<th>Hammer crusher</th>
<th>Ball mill</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( a_1 )</td>
<td>( a_2 )</td>
</tr>
<tr>
<td>Bantli</td>
<td>1.01</td>
<td>0.45</td>
</tr>
<tr>
<td>Dereboyu</td>
<td>0.98</td>
<td>0.45</td>
</tr>
<tr>
<td>Kef</td>
<td>1.04</td>
<td>0.48</td>
</tr>
<tr>
<td>Lasir</td>
<td>1.13</td>
<td>0.43</td>
</tr>
<tr>
<td>Yunuskuyu</td>
<td>1.18</td>
<td>0.39</td>
</tr>
</tbody>
</table>

http://www.ppsc-journal.com © 2009 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim
the data in the two scaling regions for all studied chromite samples.

4.4 Overall Performance Evaluation of PSD Models

To assess the performance of the PSD distribution functions with different types of chromite ore samples for the entire range of data for each comminution device, linear regression analyses between predicted and observed mass values using the PSD models (GGS model, RR model and piecewise approach to GGS model) were carried out. The observed data was the percent mass measured during sieving while the predicted data was the amounts found by the models. Table 9 shows the results (predicted versus observed) of applying GGS, RR and piecewise distribution models to the PSDs of chromite samples and compares the performance of the models. All pairs of observed and predicted values obtained for each chromite samples in each comminution device were included in the regression models. The number of observations (n) was obtained by multiplying the number of samples (5 ore samples) by the number the sieve fractions (for example, for jaw crusher: 5 samples × 18 sieve fractions = 90 observations).

The average $R^2$ for GGS and RR models were 0.986 and 0.974 for jaw crushing, 0.976 and 0.95 for the first stage of cone crushing and 0.992 and 0.978 for the second stage of cone crushing, respectively, when the different mineralogical samples of chromite ores were comminuted by low-energy events and the fracture mode was mainly due to cleavage. The comparisons showed that the GGS function was best at describing the jaw and cone crusher’s data, when looking at standard errors of regression coefficients for the fitted values. When the ore samples were subjected to high-energy events where the main breakage mode was shattering, the average values of $R^2$ for GGS and RR functions were found to be 0.947 and 0.985 for the hammer crusher and 0.952 and 0.983 for the ball mill, respectively (Table 9). The RR function was best at describing the hammer crusher and ball mill data in terms of model predicted values versus experimentally predicted values. In work reported previously, the RR model was found to be a good estimate of the cumulative PSD of ground products from stirred media mills [12]. In many grinding cases, mainly during the last stages of the process, the cumulative particle size distribution was found to follow the RR equation [26]. For impact fragmentation, Grady and Kipp suggested the RR representation to characterize the cumulative distribution of fragment fractions [27].

The relationship between distribution modulus of both models and $R^2$ is shown in the Figure 7. The figure suggests that both size distribution models were sensitive to the spread of size distribution. The trend lines in Figure 7 show that both distribution models fits the data best at higher values of $a$ and $b$ (narrower size distributions). Increasing of range of size distribution has a reverse effect on the efficiency of the GGS function, than RR model. The results implied that the RR model may be more suitable for evaluating PSDs which are more scattered and in wider size distributions than the GGS model. Consistent with the literature [28], its efficiency also increases with increasing $b$, i.e., the size distribution become more uniform.

The distribution modulus of both distribution models, $a$ and $b$, which describe the fineness and the size range of the PSD, properly identify the differences between the ores comminuted in each device. The distribution parameters of both models separated the chromite ores into similar groups. As can be clearly seen from Figure 8, there was a close relationships between these parameters, as $a$ increased, $b$ also increased and suggested the same results.

The highest $R^2$ values were obtained for all types of chromite ores when a piecewise linear regression was applied to the GGS model of hammer crusher and ball mill products. 99.8% of the total variation in the observed data values were explained by using piecewise linear regression. Harris [29] suggested dividing the PSD into two regions, fine and coarse, with each region described by a specific distribution when a single law cannot describe the data across the entire range of measured sizes. Consistent with this suggestion, the piecewise lin-
ear regression improved the performance of GGS distribution in terms of correlation coefficient for samples from the hammer crusher and ball mill.

5 Conclusions

Depending on the dominant breakage modes and initial mineralogical characteristics and size of the ore particles, different PDSs for chromite ore samples were obtained by different comminution devices that were operated in their conventional working conditions and characterized by using GGS and RR models.

The distribution modulus of GGS and RR functions obtained for chromite ore samples offered a characteristic range of values in the same comminution device. This results suggests that the PSDs produced by comminution therefore depends to a large extent on the mode of fragmentation used which has an important influence on particle-size distribution of the resulting fragments, and to a lesser extent on the ore characteristics.

Although each of the PSD models offered enough flexibility to properly describe PSDs for different mineralogical samples of chromite ores, the GGS model should be considered for characterizing of PSDs generated by low-energy events, i.e., jaw and cone crushing while the RR model was more suitable for PSDs obtained by high-energy events, i.e., hammer crushing and ball milling.

The efficiency of the both distribution models was found to increase as the size distribution got narrower. The GGS function was more reversely affected by the spread of the distribution than the RR function.

The results of piecewise regression indicated that GGS distribution plots of hammer crusher and ball mill products showed the presence of two different scaling domains for different ranges of particle sizes. The presented model was better than traditional distributions (GGS and RR) in terms of $R^2$ since higher correlation coefficients were obtained than the other PSD models when a piecewise linear regression approach was applied.

It can be concluded that particle size and distribution can usually be related to a particular mode of breakage, the mineralogical properties of the ores comminuted, comminution time and feed size, etc., under the usual working conditions of the devices. Altering the grinding environment may change the relative proportion of energy events and consequently alter the PSDs of the ground products [12]. Therefore, whether or not the PSDs obtained in altered conditions can be characterized properly in all situations by the application of a single distribution function may be examined further.

6 Nomenclature

- $a$ – distribution parameter of the GGS function
- $b$ – spread of distribution by the RR function
- GGS – Gates–Gaudin–Schuhmann function
- $k$ – size parameter of the GGS function
- $M$ – cumulative mass passing size, $x$
- PSD – particle size distribution
- $R$ – cumulative mass retaining size, $x$
- RR – Rosin–Rammelr function
- $x_c$ – size parameter of piecewise linear regression
- $x'$ – size parameter of RR function

7 References


